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5–11 Using Several Methods of Factoring

Objective: To factor polynomials completely.

Vocabulary

Factored completely A polynomial expressed as the product of a monomial and one or more prime polynomials, that is when it cannot be factored further.

Guidelines for Factoring Completely

1. Factor out the greatest monomial factor first.
2. Look for a difference of squares.
   Pattern: $a^2 - b^2 = (a - b)(a + b)$ (However, $a^2 + b^2$ can't be factored.)
3. Look for a perfect square trinomial.
   Patterns: $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$
4. If the trinomial is not a perfect square, look for a pair of binomial factors.
5. If a polynomial has four or more terms, look for a way to group the terms in pairs or in a group of three terms that is a perfect square trinomial.
6. Make sure that each binomial or trinomial factor is prime.
7. Check your work by multiplying the factors.

Example 1 Factor $8x^3 - 512x$ completely.

Solution $8x^3 - 512x = 8x(x^2 - 64)$

Greatest monomial factor: $8x$; Difference of squares: $x^2 - 64$

$= 8x(x + 8)(x - 8)$

Example 2 Factor $3x^3 + 3x^2 - 18x$ completely.

Solution $3x^3 + 3x^2 - 18x = 3x(x^2 + x - 6)$

Greatest monomial factor: $3x$; Trinomial: $x^2 + x - 6$

$= 3x(x + 3)(x - 2)$

Factor completely.

1. $3x^3 - 12x$
2. $5m^3 - 45m$
3. $3a^2 + 6ab + 3b^2$
4. $-x^3 + 4xy^2$
5. $-12z^3 + 30z^2 + 18z$
6. $16r^4 - 24r^3 + 9r^2$
7. $20x^3 - 28x^2 + 8x$
8. $t^3 + t^2 - 2t$
9. $2x^2 - 128$
10. $2x^4 - 162$
11. $25z^3 - 36y^2z$
12. $6x^2 + 22xy - 8y^2$

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5–11 *Using Several Methods of Factoring* *(continued)*

Example 3 Factor $5a^2b^3 + 2a^3b^2 - 3ab^4$ completely.

Solution First rewrite the polynomial in order of decreasing degree in a.

$$5a^2b^3 + 2a^3b^2 - 3ab^4 = 2a^3b^2 + 5a^2b^3 - 3ab^4$$
$$= ab^2(2a^2 + 5ab - 3b^2)$$

Greatest monomial factor: ab^2; Trinomial: $2a^2 + 5ab - 3b^2$

$$= ab^2(2a - b)(a + 3b)$$

Example 4 Factor $a^2b - 4b + 3a^2 - 12$ completely.

Solution

$$a^2b - 4b + 3a^2 - 12 = b(a^2 - 4) + 3(a^2 - 4)$$ Group and factor.
$$= (b + 3)(a^2 - 4)$$ Use the distributive property.

Difference of squares: $a^2 - 4$

$$= (b + 3)(a + 2)(a - 2)$$

Factor completely.

13. $a^3x - 9ax^3$
14. $18x^3 - 24x^2 + 8x$
15. $20 - 60x + 45x^2$
16. $6x^2 - 18xy + 12y^2$
17. $9x^3 + 108x + 63x^2$
18. $10k^3 + 25k - 35k^2$
19. $32r^4 - 48r^3 + 18r^2$
20. $12ab - 3b^2 - 12a^2$
21. $bc^2 - 4b + 3c^2 - 12$
22. $x^3 - x + 6x^2 - 6$
23. $x^2 + 6xy + 9y^2 - 16$
24. $a^3 + a^2b - ab^2 - b^3$
25. $y^4 - 9y^2 + 20$
26. $x^4 - 10x^2 + 9$
27. $x^4 - 13x^2 + 36$
28. $x^4 - 24x^2 + 144$
29. $b^4 - 8b^2 + 16$
30. $a^3 + 2a^2 - 5a - 10$

Mixed Review Exercises

Simplify.

1. $\left(-\frac{1}{3}\right)\left(\frac{1}{4}\right)(60)$
2. $\frac{1}{8}(56)$
3. $-\frac{1}{7}(56)\left(-\frac{1}{8}\right)$
4. $\frac{120b}{8}$
5. $45 \div \left(\frac{1}{5}\right)$
6. $600 \div (-5)$

Factor.

7. $x^2 - 11x + 30$
8. $x^2 + 2x - 35$
9. $x^2 - x - 20$
10. $2n^2 + 15n + 7$
11. $3x^2 + 7x + 4$
12. $(2x - 6) - 3n(3 - x)$